ELEC 342 Lab 1
AC/DC Circuits and Basic Measurements

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Lab Experiment: 1
Section: L2C
Bench: #7

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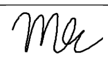
Manav Khangura	16725475	33.3 %	
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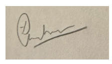
Yuvraj Chadha	23005697	33.3 %	

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Pre-Lab

Warrick Lo, 09 February 2026

Using $S = P + jQ = VI$, we get

$$S = I^2 Z = I^2 \left(R + jX\right) \quad \text{and}$$
$$S = \frac{V^2}{Z} = \frac{V^2}{R + jX}.$$

Thus, we can solve for P and Q when the impedances are all either purely resistive or reactive:

$$P = I^2 R, \tag{1a}$$
$$= \frac{V^2}{R}, \quad \text{and} \tag{1b}$$
$$Q = I^2 X, \tag{1c}$$
$$= \frac{V^2}{X}. \tag{1d}$$

Using the power triangle and the definition of the power factor angle,

$$\cos\varphi = \frac{P}{S}, \quad \tan\varphi = \frac{Q}{P},$$

and thus,

$$Q = P \tan\varphi. \tag{2}$$

Resistance

Series resistance:

$$R = \frac{P}{I^2} \qquad \text{Using equations (1a) and (2)} \tag{3a}$$

Parallel resistance:

$$R = \frac{V^2}{P} \qquad \text{Using equations (1b) and (2)} \tag{3b}$$

Inductance

$$jX_L = j\omega L$$
$$L = \frac{X_L}{\omega} \tag{4}$$

Series inductance:

$$\begin{aligned} L &= \frac{Q}{\omega I^2} && \text{Using equations (1c) and (4)} \\ &= \frac{P\tan\varphi}{\omega I^2} && \text{Using equation (2)} \\ &= \frac{P\tan\varphi}{2\pi f I^2} && (5a) \end{aligned}$$

Parallel inductance:

$$
\begin{aligned}
L &= \frac{V^2}{\omega Q} && \text{Using equations (1d) and (4)} \\
&= \frac{V^2}{\omega P \tan\varphi} && \text{Using equation (2)} \\
&= \frac{V^2}{2\pi f P \tan\varphi}
\end{aligned} \tag{5b}
$$

Capacitance

$$
\begin{aligned}
-\mathrm{j}X_C &= \frac{1}{\mathrm{j}\omega C} = \frac{-\mathrm{j}}{\omega C} \\
C &= \frac{1}{\omega X_C}
\end{aligned} \tag{6}
$$

Series capacitance:

$$
\begin{aligned}
C &= \frac{I^2}{\omega Q} && \text{Using equations (1c) and (6)} \\
&= \frac{I^2}{\omega P \tan\varphi} && \text{Using equation (2)} \\
&= \frac{I^2}{2\pi f P \tan\varphi}
\end{aligned} \tag{7a}
$$

Parallel capacitance:

$$
\begin{aligned}
L &= \frac{Q}{\omega V^2} && \text{Using equations (1d) and (6)} \\
&= \frac{P \tan\varphi}{\omega V^2} && \text{Using equation (2)} \\
&= \frac{P \tan\varphi}{2\pi f V^2}
\end{aligned} \tag{7b}
$$

Instantaneous Power

$$
\cos\alpha \cos\beta = \frac{1}{2}\left(\cos(\alpha+\beta) + \cos(\alpha-\beta)\right) \tag{8}
$$

$$
\begin{aligned}
v(t) &= \sqrt{2} V_{\mathrm{RMS}} \cos(\omega t + \varphi_v) \\
i(t) &= \sqrt{2} I_{\mathrm{RMS}} \cos(\omega t + \varphi_i)
\end{aligned}
$$

$$
\begin{aligned}
p(t) &= v(t) i(t) \\
&= 2 V_{\mathrm{RMS}} I_{\mathrm{RMS}} \cos(\omega t) \cos(\omega t + \varphi) \\
&= V_{\mathrm{RMS}} I_{\mathrm{RMS}} \left(\cos\varphi + \cos(2\omega t + \varphi)\right)
\end{aligned} \tag{9}
$$

Complex Power

$$
\begin{aligned}
S &= P + \mathrm{j}Q \\
&= VI\cos\varphi + \mathrm{j}VI\sin\varphi
\end{aligned}
$$

$$
P = VI\cos\varphi \tag{10a}
$$

$$
Q = VI\sin\varphi \tag{10b}
$$

Three-Phase Power

$$V_{\text{line}} = \sqrt{3} V_{\text{phase}}$$

$$\begin{aligned} P &= 3 V_{\text{phase}} I_{\text{phase}} \cos\varphi && \text{(11a)} \\ &= \sqrt{3} V_{\text{line}} I_{\text{line}} \cos\varphi && \text{(11b)} \end{aligned}$$

$$\begin{aligned} Q &= 3 V_{\text{phase}} I_{\text{phase}} \sin\varphi && \text{(12a)} \\ &= \sqrt{3} V_{\text{line}} I_{\text{line}} \sin\varphi && \text{(12b)} \end{aligned}$$

Task 1A. Measurements with RL Load Box

Measurement	$V_{1(\text{RMS})}$ (V)	$I_{1(\text{RMS})}$ (A)	φ_1 (°)	$P_{1(\text{AVG})}$ (W)	Z_{eq} (Ω)	$R_{\text{eq(series)}}$ (Ω)	L_{eq} (mH)
One inductor	14.11	0.64	85.4	0.72	22.18	1.758	57.95
Three inductors	14.07	1.98	85.2	2.33	7.11	0.594	18.77

Table 1. Single-phase parallel inductors.

- **How is this inductor different from an ideal inductor element?**
  The inductor has many parasitic elements, including physical resistance from the metal conductor, self-capacitance, magnetic flux leakage, radiation losses at high frequencies, and core losses if it's not an air-core inductor.
- **Are the calculated values of inductance close to the expected value? What is the difference?**
  The nominal value for each inductor, as indicated on the load box, is 40 mH. With three inductors in parallel, this gives us an equivalent inductance of 13.33 mH. The measured values are higher, at 57.95 mH for the single inductor and 18.77 mH for the parallel inductors, giving a deviation of 45 % and 41 %, respectively. This is a relatively high amount of deviation and can be attributed to the parasitic properties of the inductor described above.

Task 1B. Measurements with RC Load Box

Measurement	$V_{1(\text{RMS})}$ (V)	$I_{1(\text{RMS})}$ (A)	φ_1 (°)	$P_{1(\text{AVG})}$ (W)	Z_{eq} (Ω)	$R_{\text{eq(parallel)}}$ (Ω)	C_{eq} (μF)
One capacitor	14.07	0.22	-89.6	0.022	62.82	0.4545	40.74
Three capacitors	14.12	0.68	-89.8	0.034	20.47	0.0735	125.9

Table 2. Single-phase parallel capacitors.

- **How is this capacitor different from an ideal capacitor element?**
  The capacitor has physical resistance from the metal conductor, self-inductance, fringing field losses, leakage current through the dielectric, and the skin effect at high frequencies, among other losses.
- **Are the calculated values of capacitance close to the expected value? What is the difference?**
  The nominal value for each capacitor, as indicated on the load box, is 40 μF. With three capacitors in parallel, the equivalence capacitance is 120 μF. The measured values are slightly higher, at 40.74 μF for the single capacitor and 125.9 μF for the parallel capacitors. This gives us a deviation of 1.9 % and 4.9 %, respectively, which is a reasonably low amount of deviation that can be attributed to the real-world properties of the capacitor described earlier.

Task 1C. Parallel Connection of RLC Components

Measurement	$I_{1(\text{RMS})}$ (A)	φ_1 (°)	$I_{2(\text{RMS})}$ (A)	φ_2 (°)	$I_{3(\text{RMS})}$ (A)	φ_3 (°)	$P_{1(\text{AVG})}$ (W)
R_1	0.71	0.2	—	—	—	—	10.0
R_1 and L_1	0.99	39.8	0.64	85.2	—	—	10.7
R_1, L_1, C_1, C_2, C_3	0.79	-1.0	0.63	85.1	0.67	-89.5	10.7

Table 3. Parallel connection of RLC components. Supply voltage $V_1 = 14.16\measuredangle - 1.7°$ V RMS.

- **How has the real power changed when you connected inductors and capacitors in parallel?**
  In the ideal scenario, adding inductors and capacitors to the circuit wouldn't change the real power as the components are purely reactive. However due to the parasitic resistances of the components, they consume small amounts of real power. The total real power of the circuit is then simply the sum of the resistor's real power and the reactive components' real power from losses.

- **How are the magnitudes of the currents I_1, I_2, and I_3 related to each other when all elements are connected in parallel?**
  The current I_1 is the sum of all three branch currents as phasors:

  $$\tilde{I}_1 = \tilde{I}_R + \tilde{I}_L + \tilde{I}_C.$$

  In the ideal model, the inductor and capacitor currents are, respectively, 90° and −90° out of phase from the resistor current. Since they are 180° apart, the magnitudes will subtract from each other. Thus the magnitude of the current I_1 can be given by

  $$|\tilde{I}_1| = \sqrt{|\tilde{I}_R|^2 + \left(|\tilde{I}_2| - |\tilde{I}_3|\right)^2}. \tag{13}$$

- **Can either I_2 and/or I_3 have a magnitude larger than the magnitude of I_1? Explain.**
  Yes, if the resistor current is small enough, or equally if both inductor and capacitor currents are large enough. We can see this in equation (13); the reactive components' currents "nearly cancel" each other out, giving only a small increase to I_1, but they can each be larger than I_1 individually.

Task 1D. Series Connection of RLC Components

Measurement	$I_{1(\text{RMS})}$ (A)	φ_1 (°)	$V_{2(\text{RMS})}$ (V)	φ_2 (°)	$V_{3(\text{RMS})}$ (V)	φ_3 (°)	$P_{1(\text{AVG})}$ (W)
R_1, L_1, C_1	0.32	55.3	8.65	218.3	20.58	31.2	2.3
R_1, L_1, C_1, C_2, C_3	0.64	0.8	14.25	80.6	13.97	90.5	9.0
$R_1, R_2, R_3, L_1, C_1, C_2, C_3$	1.56	25.3	27.17	245.0	34.08	60.6	19.5

Table 4. Series connection of RLC components. Supply voltage $V_1 = 14.14\measuredangle - 1.6°$ V RMS.

- **How has the real power changed when you connected capacitors in parallel or resistors in parallel?**
  When the capacitors are connected in parallel, the equivalent impedance of the capacitors is significantly lowered. This matches with how see V_3 being lower afterwards. Thus, more current is flowing through the circuit and more power is being dissipated by the resistor. When the resistors are connected in parallel, more current is again going through the circuit and thus the real power increases as well.

- **How are the magnitudes of voltages V_1, V_2, and V_3 related to each other when all elements are connected in series?**
  The voltage V_1 is the sum of all three components' voltages as phasors:

  $$\tilde{V}_1 = \tilde{V}_R + \tilde{V}_L + \tilde{V}_C.$$

  In the ideal model, the inductor and capacitor voltages are, respectively, 90° and −90° out of phase from the resistor voltage. Since they are 180° apart, the magnitudes will subtract from each other. Thus the magnitude of the voltage V_1 can be given by

  $$|\tilde{V}_1| = \sqrt{|\tilde{V}_R|^2 + \left(|\tilde{V}_2| - |\tilde{V}_3|\right)^2}. \tag{14}$$

- **Can either V_2 and/or V_3 have a magnitude larger than the magnitude of V_1? Explain.**
  Yes, if the resistor voltage is small enough, or equally if both inductor and capacitor voltages are large enough. Similar to Task 1C, we can see how according to equation (14), the reactive components' voltages "nearly cancel" each other out, which allows V_2 and V_3 to be each larger than V_1 individually.

- **Support your answer with the phasor diagram corresponding to the measurement Task 1D, step 5.**
  See figure 1.

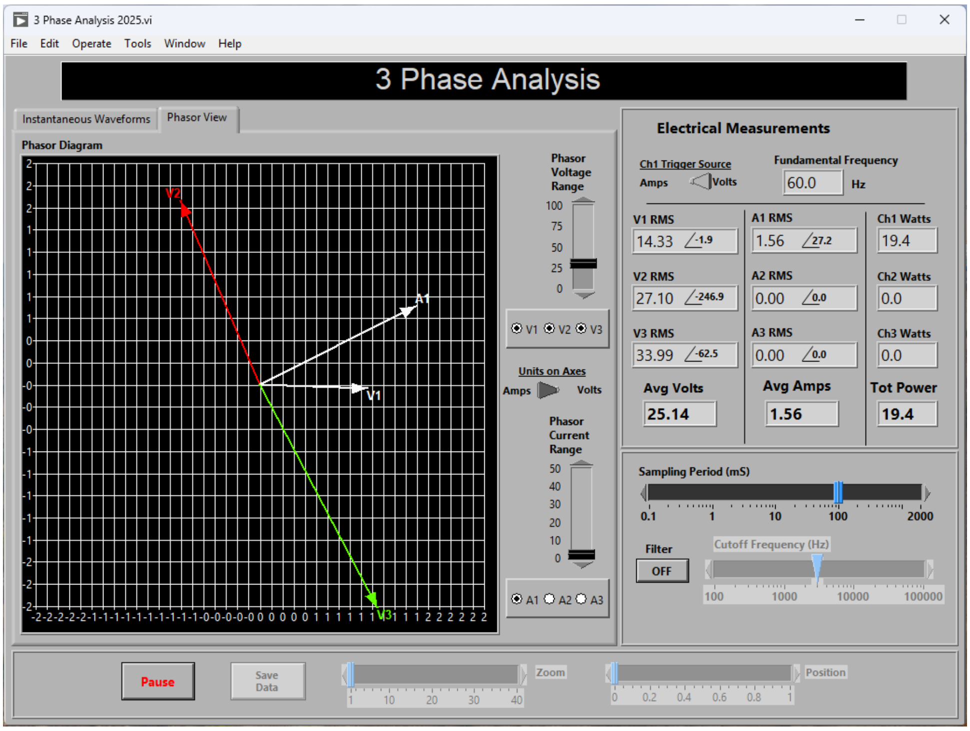


Figure 1. Phasor view of circuit with three resistors, one inductor, and three capacitors.

Task 2A. Three-Phase Measurements with Wye-Connected RL Load Box

	Phase 1 (RMS)		Phase 2 (RMS)		Phase 3 (RMS)		Total
Measurement	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	P_{AVG} (W)
RL balanced load, neutrals disconnected	10.17 ∡ 4.5	0.27 ∡ -46.3	10.15 ∡ -238.0	0.29 ∡ 73.3	9.79 ∡ -115.2	0.29 ∡ -163.6	5.6
RL unbalanced load, shorted resistor, neutrals disconnected	10.06 ∡ 4.4	0.38 ∡ -80.0	12.91 ∡ -251.8	0.39 ∡ 60.8	8.94 ∡ -139.9	0.26 ∡ -189.0	5.2
RL unbalanced load, shorted inductor, neutrals disconnected	8.88 ∡ 0.9	0.45 ∡ 0.6	9.15 ∡ -199.3	0.26 ∡ -247.8	13.71 ∡ -98.4	0.43 ∡ -145.0	9.9
RL unbalanced load, shorted resistor, neutrals connected	10.16 ∡ -1.5	0.4 ∡ -85.7	10.31 ∡ -241.4	0.3 ∡ 70.2	10.19 ∡ -120.6	0.30 ∡ -168.8	4.6
RL unbalanced load, shorted inductor, neutrals connected	10.09 ∡ -1.8	0.51 ∡ -2.0	10.27 ∡ -241.5	0.30 ∡ 70.3	10.19 ∡ -120.8	0.30 ∡ -168.9	9.6

Table 5. Balanced and unbalanced wye-connected RL three-phase load.

- **How close are your measurements to the ideal case when the RL load is balanced?**
  In the balanced load circuit, the voltages had phase differences that were calculated to be 120.0° and maximum voltage spread of less than ±0.38 V. Similarly, the currents had phase differences of 120.0° and variations in magnitude of ±0.02 A. These can be attributed to the tolerances in individual components, manufacturing variations, and parasitic impedances, among other things.

- **What happens to the phase voltages and currents when the resistor is shorted in one of the phases?**
  Looking at the phase current through the shorted load, the magnitude will increase due to the lower impedance from the short. That current will also lag the voltage by around 90°, due to it being a (nearly) pure inductive load. Without the neutral line connected, the phase voltages drift from the target of 10 V and the phases deviate from their nominal 120° separation.
- **What happens to the phase voltages and currents when the inductor is shorted in one of the phases?**
  Looking at the phase current through the shorted load, the magnitude will again increase due to the lower impedance from the short. That current will be roughly in phase with the voltage, due to it being purely resistive. Without the neutral line connected, the phase voltages also drift from the target of 10 V and the phases again deviate from their nominal 120° separation.
- **How does connecting or disconnecting the neutral wire affect the phase voltages and currents when the load is unbalanced?**
  The neutral line allows the unbalanced current to flow back to the source. This stabilises the phase voltages even when the load is unbalanced, ensuring that the magnitudes stay around 10 V and the phase differences are around 120°.

Task 2B. Three-Phase Measurements with Delta-Connected RL Load Box

	Phase 1 (RMS)		Phase 2 (RMS)		Phase 3 (RMS)		Total
Measurement	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	P_{AVG} (W)
RL balanced load	9.98 ∡ -1.6	0.93 ∡ -47.0	10.06 ∡ -241.9	0.95 ∡ 72.0	9.93 ∡ -121.3	0.96 ∡ -167.1	19.7
RL unbalanced load, shorted resistor	10.10 ∡ -1.5	0.94 ∡ -81.0	10.14 ∡ -241.7	1.39 ∡ 55.0	10.02 ∡ -121.3	0.97 ∡ -167.2	14.6
RL unbalanced load, shorted inductor	9.97 ∡ -1.5	1.41 ∡ -26.2	10.15 ∡ -242.3	0.91 ∡ -248.7	10.03 ∡ -120.9	0.97 ∡ -166.6	28.8

Table 6. Balanced and unbalanced delta-connected RL three-phase load.

- **How close are your measurements to the ideal case when the RL load is balanced?**
  In the balanced load circuit, the voltages had phase differences that were calculated to be 120.0° and maximum voltage spread of less than ±0.13 V. Similarly, the currents had phase differences of 120° and variations in magnitude of ±0.03 A. These can be attributed to the tolerances in individual components, manufacturing variations, and parasitic impedances, among other things.
- **How do the current and power measurements (values) compare with the previous case when the balanced load was wye-connected?** The average power increases by a factor of 3, compared to the wye-connected load. This is because the power per phase in a wye-connected load is given by $\left(V_{\text{line}}/\sqrt{3}\right)^2/Z\cdot\cos\varphi$, whereas the delta-connected load's power per phase is given by $V_{\text{line}}^2/Z\cdot\cos\varphi$. Similarly, the line current will increase by a factor of 3, since in wye and delta connections the line current is given by $V_{\text{line}}/\left(\sqrt{3}Z\right)$ and $\sqrt{3}V_{\text{line}}/Z$, respectively.
- **What happens to the phase voltages and currents when the resistor is shorted in one of the phases?** The phase voltages do not change, staying approximately 120° apart and having a maximum voltage spread of 0.12 V. Looking at the phase current through the shorted load, the magnitude will increase due to the lower impedance from the short. That current will also lag the voltage by around 90°, due to it being a (nearly) pure inductive load.
- **What happens to the phase voltages and currents when the inductor is shorted in one of the phases?** The phase voltages do not change, staying approximately 120° apart and having a maximum voltage spread of 0.18 V. Looking at the phase current through the shorted load, the magnitude will again increase due to the lower impedance from the short. That current will be in phase with the voltage, due to it being a purely resistive load.

Task 2C. Two-Wattmeter Measurement Method with Delta-Connected RL Load Box

	Phase 1 (RMS)		Phase 2 (RMS)		Average Power		
Measurement	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	$V\measuredangle\varphi_v$ (V)	$I\measuredangle\varphi_i$ (A)	P_1 (W)	P_2 (W)	P_{total} (W)
RL balanced load	17.34 ∡ -0.7	0.94 ∡ -76.2	17.58 ∡ 59.4	0.96 ∡ 42.6	4.1	15.6	19.8
RL unbalanced load, shorted resistor	17.41 ∡ -0.7	0.94 ∡ -110.3	17.53 ∡ 59.5	1.40 ∡ 25.7	-5.5	19.7	14.2
RL unbalanced load, shorted inductor	17.37 ∡ -0.6	1.42 ∡ -55.6	17.67 ∡ 59.1	0.92 ∡ 81.8	14.3	14.5	28.7

Table 7. Two-wattmeter measurements with delta-connected RL three-phase load.

- **The load circuit in Task 2B and Task 2C is identical in each step. Compare the measurements of currents and voltages of Task 2B and Task 2C. Explain the results.**
  The voltages measured here are greater compared to Task 2B because we are measuring the line-to-line voltage, as opposed to the phase voltage like what was done previously. As the line and phase voltages are related to each other by $V_{\text{line}} = \sqrt{3}V_{\text{phase}}\measuredangle 30°$, the voltage supply is giving the same output voltage in both Task 2B and Task 2C (as $\sqrt{3} \cdot 10\,\text{V} \cong 17.32\,\text{V}$). We also see this through our measurement of the current; the magnitude remains relatively unchanged from Task 2B to Task 2C, while the current shows a $-30°$ phase shift because of the change in our reference (the angle of V_1).
- **Compare the measurements of the real power of Task 2B and Task 2C. Explain the results.**
  The total real power remains relatively the same, as expected, since no part of the physical circuit changed. We see that when the resistor is shorted (the load is purely inductive), P_1 is negative. This can be explained by the power factor angle being more than 90°, which will make the power, given by $P = V_{\text{line}}I_{\text{phase}}\cos\varphi$, negative.

Task 3A. Single-Phase Full-Wave Rectifier

	$V_{1(\text{RMS})}$ (V)	$I_{1(\text{RMS})}$ (A)	$P_{1(\text{in})}$ (W)	$V_{3(\text{RMS})}$ (V)	$V_{3(\text{PP})}$ (V)	$I_{3(\text{RMS})}$ (A)	$I_{3(\text{PP})}$ (A)	$P_{3(\text{out})}$ (W)
No load	15.05 ∡ -1.7	0.08 ∡ 36.2	0.8	14.19 ∡ -93.8	20.5	0.11 ∡ -224.5	0.4	1.0
Light load	14.87 ∡ -1.8	0.84 ∡ -2.4	12.5	13.34 ∡ -93.9	20.6	0.88 ∡ -96.2	0.9	11.7
Heavy load	14.77 ∡ -1.8	1.50 ∡ -2.0	22.0	13.09 ∡ -94.1	20.2	1.53 ∡ -95.6	2.6	19.9

Table 8. Single-phase full-wave rectifier, without a capacitor filter.

- **What relationship between the voltage and current peak/ripple do you observe?**
  The current follows the same waveform shape as the voltage. Since the load is resistive and there is no capacitor, the current peak is in phase with the voltage peak. As the load resistance decreases (heavier load), the peak to peak current increases while the voltage peak to peak remains about constant.

	$V_{1(\text{RMS})}$ (V)	$I_{1(\text{RMS})}$ (A)	$I_{1(\text{PP})}$ (A)	$P_{1(\text{in})}$ (W)	$V_{3(\text{RMS})}$ (V)	$V_{3(\text{PP})}$ (V)	$I_{3(\text{RMS})}$ (A)	$I_{3(\text{PP})}$ (A)	$P_{3(\text{out})}$ (W)
No load	15.04 ∡ -1.7	0.34 ∡ -8.6	2.8	2.2	19.90 ∡ -176.3	0.29	0.17 ∡ 13.1	0.16	2.8
Light load	14.73 ∡ -1.8	2.5 ∡ -3.9	13.5	24.8	17.73 ∡ -165.8	1.58	1.23 ∡ 203.1	0.23	21.7
Heavy load	14.54 ∡ -2.1	3.80 ∡ -2.3	18.5	39.4	16.73 ∡ -161.9	2.30	2.01 ∡ -159.5	0.36	33.6

Table 9. Single-phase full-wave rectifier, with a capacitor filter.

- **How do the output voltage and current ripples compare with the previous case without the capacitor filter?**
  Compared to the no-capacitor case, adding the capacitor filter reduces both the output voltage and current ripple,

because the capacitor supplies current between rectified peaks. Ripple increases with heavier load due to faster discharge, but it is smaller than without the filter.

- **How does the presence of a capacitor impact the input current peak?**
The input current peak is increased when the capacitor is present. Current flows in brief, high-amplitude pulses close to the AC peaks rather than continuously throughout the cycle because the capacitor charges only when the input voltage is greater than the capacitor voltage. In comparison to the scenario in which the capacitor filter is not used, this leads to significantly greater input current peaks.
- **How does the peak of the input current change with the load current?**
The peak input current rises noticeably as the load current does. A higher recharge current is needed over a brief interval when the load is heavier because the capacitor discharges more between AC peaks. Higher amplitude current pulses at the input are the outcome of this.

Task 4A. Three-Phase Full-Wave Rectifier

	$I_{1(\mathrm{RMS})}$ (A)	$V_{2(\mathrm{RMS})}$ (V)	$V_{2(\mathrm{PP})}$ (V)	$I_{2(\mathrm{RMS})}$ (A)	$I_{2(\mathrm{PP})}$ (A)	$P_{3(\mathrm{out})}$ (W)
No load	0.09 ∡ 33.3	12.18 ∡ -62.2	1.5	0.16 ∡ -148.3	0.14	1.5
Light load	0.65 ∡ 33.3	11.99 ∡ -50.1	1.5	0.86 ∡ -36.7	0.22	9.9
Heavy load	1.10 ∡ 34.2	11.86 ∡ -40.6	1.5	1.43 ∡ -32.2	0.30	16.2

Table 10. Three-phase full-wave rectifier, without a capacitor filter.

- **What relationship between the voltage and current peak/ripple do you observe?**
The ripples are the same shape and in phase as the load is purely resistive. The voltage peak to peak remains constant while the current peak to increases.
- **How does the input current differ from that of the single-phase rectifier?**
The input current of a three-phase rectifier is smoother and more continuous than that of a single-phase rectifier. In the single-phase case, current flows in large pulses and drops to zero, in the three-phase case, conduction overlaps between phases, reducing ripple.

	$I_{1(\mathrm{RMS})}$ (A)	$I_{1(\mathrm{PP})}$ (A)	$I_{2(\mathrm{RMS})}$ (A)	$I_{2(\mathrm{PP})}$ (A)	$V_{3(\mathrm{RMS})}$ (V)	$V_{3(\mathrm{PP})}$ (V)	$I_{3(\mathrm{RMS})}$ (A)	$I_{3(\mathrm{PP})}$ (A)	$P_{3(\mathrm{out})}$ (W)
No load	0.08 ∡ 22.4	0.5	0.20 ∡ -31.0	0.6	12.58 ∡ -236.0	0.1	0.12 ∡ -218.5	0.1	1.6
Light load	0.70 ∡ 28.6	2.7	1.02 ∡ -54.3	2.0	11.91 ∡ -95.9	0.3	0.83 ∡ 96.0	0.1	10.0
Heavy load	1.17 ∡ 29.0	4.2	1.73 ∡ -54.4	2.7	11.63 ∡ -96.0	0.4	1.40 ∡ -183.2	0.1	16.3

Table 11. Three-phase full-wave rectifier, with a capacitor filter.

- **How do the output voltage and current ripples compare with the previous case without the capacitor filter?**
With the capacitor filter, both output voltage and current become smoother. Ripple still increases with heavier load but it remains smaller than the case without the capacitor.
- **How does the presence of a capacitor impact the input current peak?**
The input current peak is increased when the capacitor is present. Current only flows when the input voltage is higher than the capacitor voltage because the capacitor keeps the output voltage close to its maximum value. This causes short pulses as opposed to continuous conduction, which results in larger peak input currents.
- **How do the output voltage and current ripples compare with the single-phase rectifier?**
Compared to the single-phase rectifier, the three-phase rectifier produces significantly smaller output voltage and current ripple. This is because the three-phase rectifier has overlapping output between phases, which reduces the voltage dips between peaks. As a result, the output is smoother.
- **How does the peak of the input current change with the load current?**
The peak input current increases as the load current increases. A heavier load causes the capacitor to discharge more current between peaks, requiring a larger recharge current over a short time. As a result, the input current peaks become taller under heavier load conditions.

Task 5A. Instantaneous, Real, and Reactive Power in Parallel RLC Circuit

The instantaneous power is given by

$$p(t) = v(t)i(t), \tag{15}$$

where $v(t)$ and $i(t)$ are the instantaneous voltages and currents, respectively. The average power is

$$P = \frac{1}{T}\int_{\langle T\rangle} v(t)i(t)\,\mathrm{d}t. \tag{16}$$

The measured RMS voltages and currents are slightly different than the ones calculated by MATLAB, which uses the discrete version of equation (16) (see the appendix). We will use the values calculated by MATLAB here.

For a sinusoidal signal, $P = V_{\mathrm{RMS}} I_{\mathrm{RMS}} \cos\varphi$ and $Q = V_{\mathrm{RMS}} I_{\mathrm{RMS}} \sin\varphi$. We will assume that the signals are "sinusoidal enough".

	Real Power, P (W)	Reactive Power, Q (VAR)
Task 3C, step 3	9.98	0.03
Task 3C, step 4	10.68	8.90
Task 3C, step 5	11.25	-0.20

Table 12. Calculations for the instantaneous, real, and reactive power in a parallel RLC circuit. The equations for a sinusoidal signal, shown above, were used. RMS values are calculated from MATLAB.

The instantaneous power gives the power absorbed at any given moment in time, while the real power is the average power absorbed or dissipated over an interval. The reactive power is power that is temporarily stored in the magnetic and electric fields of inductors and capacitors, respectively.

By adding inductors, the reactive power increases since they have positive reactance and will increase the power factor angle. Capacitors, on the other hand, decrease the reactive power since they have negative reactive and will decrease the power factor angle.

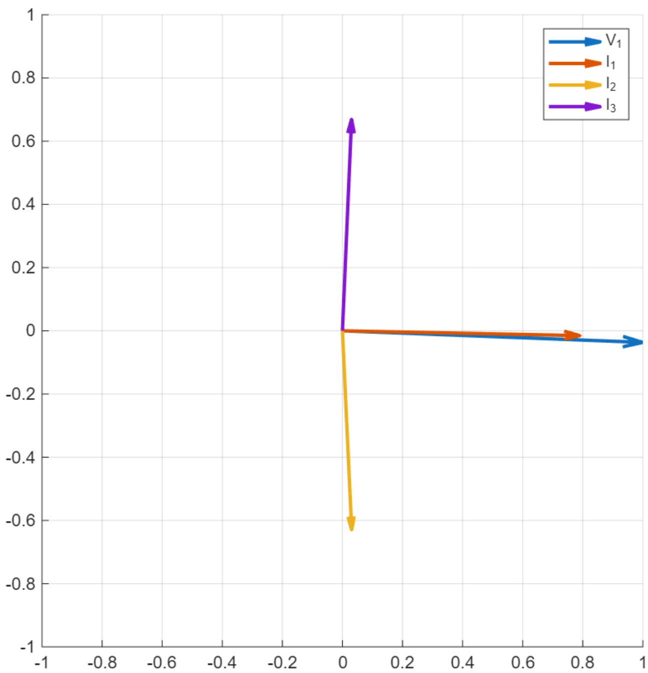
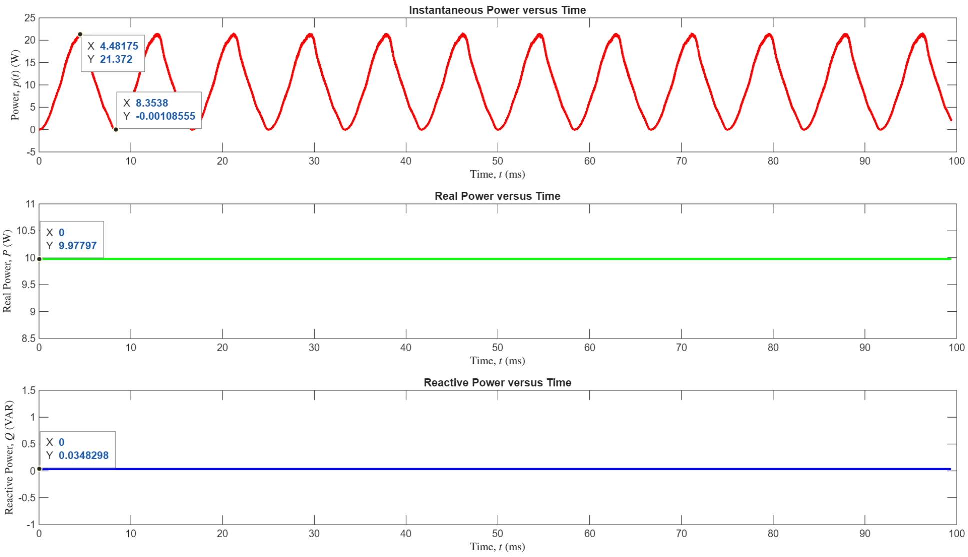
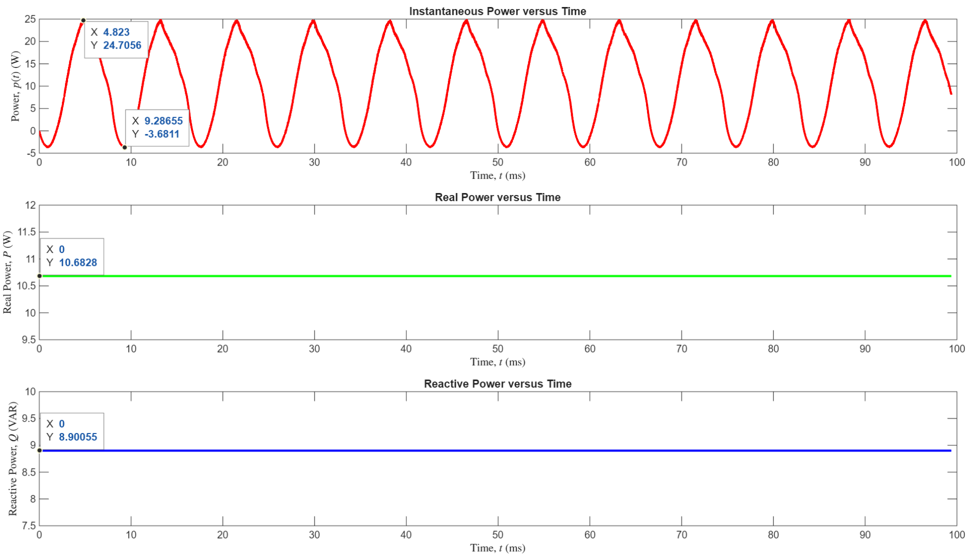


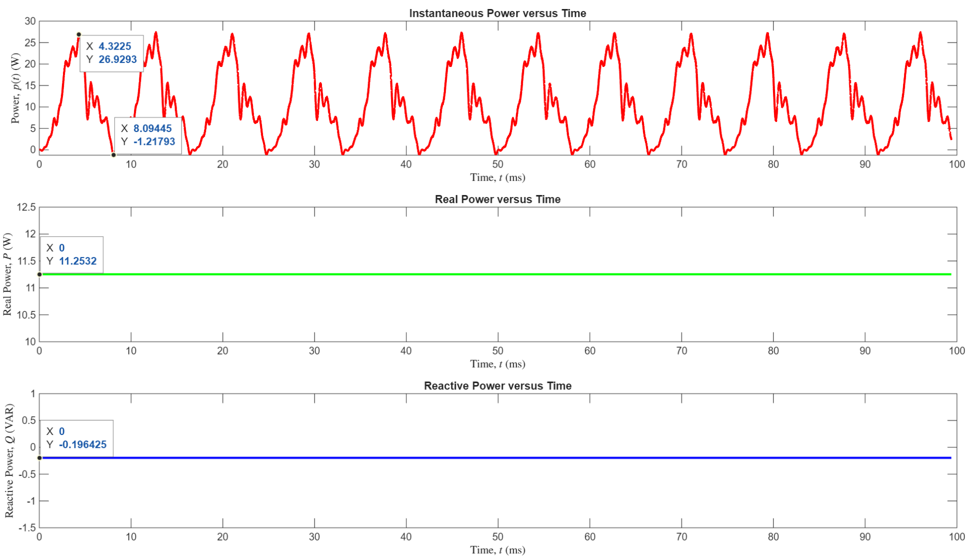
Figure 2. Phasor plot of the applied voltage V_1, input current I_1, inductor current I_2, and capacitor current I_3 for the parallel RLC circuit. Here the voltage V_1 is normalised to 1. Notice how the phasors I_2 and I_3 effectively cancel each other.



(a) Plot of circuit with one resistor.



(b) Plot of circuit with one resistor and one inductor.



(c) Plot of circuit with one resistor, one inductor, and three capacitors.

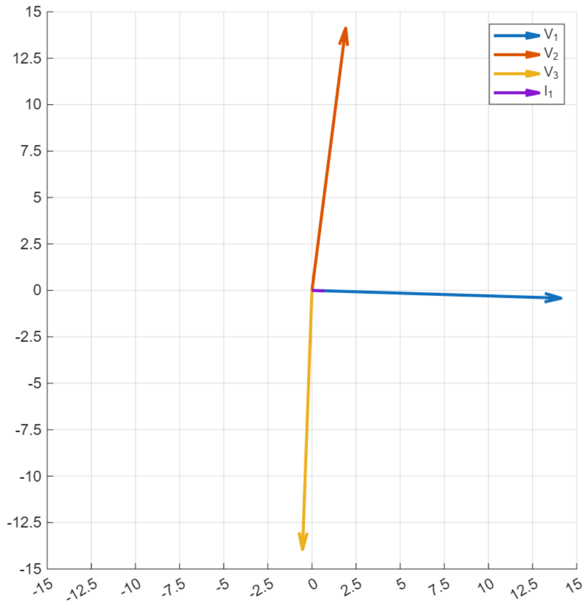
Figure 3. Plots of instantaneous, real, and reactive power in the parallel RLC circuits.

Task 5B. Single-Phase Phasor Diagram for the Series RLC Circuit

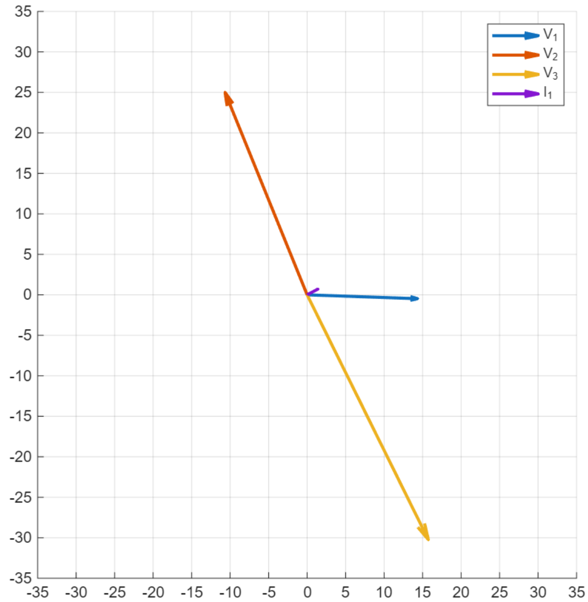
We see that in the phasor diagrams, the inductor and capacitor voltages are always 180° apart from each other, as expected since each of them have purely reactive loads. We see that with one resistor, the voltages V_2 and V_3 are almost exactly ±90° from V_1. This means that the power factor angle is near the minimum (0°) and almost all of the power delivered is real.

When we add the other resistors in parallel and decrease the series resistance, the current starts to lead the voltage, meaning that less of the total power delivered is real and more of it is reactive. Because the voltages V_2 and V_3 have to be 90° apart from the current I_1, we see that the shift in the current also causes the angles of the reactive components to shift as well.

Finally we observe that the difference between V_1 and $V_2 + V_3$ should be the voltage drop across the resistor(s).



(a) Phasor diagram of circuit with one resistor, one inductor, and three capacitors.



(b) Phasor diagram of circuit with three resistors, one inductor, and three capacitors.

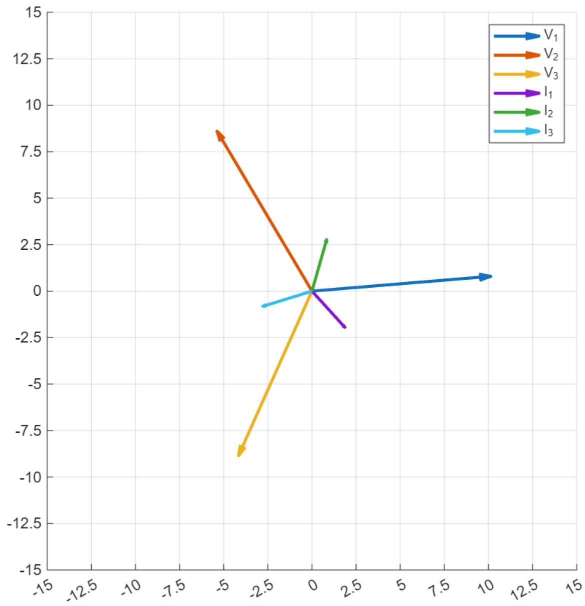
Figure 4. Phasor plots of the applied voltage V_1, inductor voltage V_2, capacitor voltage V_3, and input current I_1 for the series RLC circuit.

Task 5C. Three-Phase Phasor Diagrams for Wye-Connected RL Load

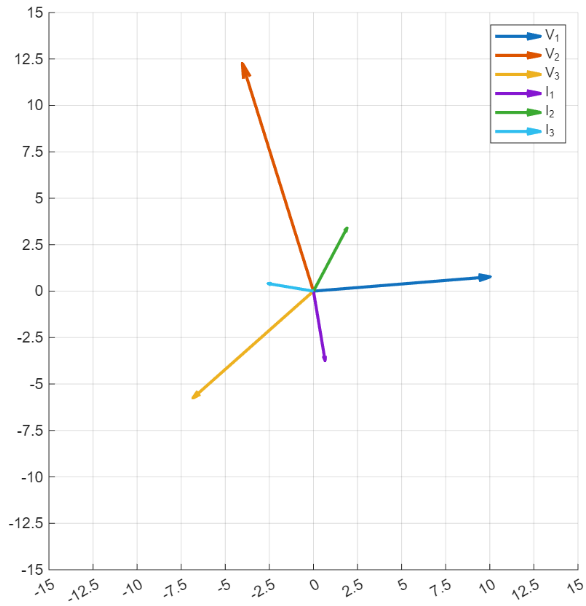
We see that without the neutral line connected, shorting one of the load elements will shift the phase voltage angles away from 120°. Shorting the resistor caused the angle of the unbalanced load's voltage to increase, and shorting the inductor caused it to decrease.

With the neutral line connected, the unbalanced current now has a return path back to the source, ensuring that the phase voltages stay relatively close to 120° apart from each other.

Looking at our measurements in table 6, we see that the unbalanced loads wouldn't affect the phase voltages, since each point of connection in the delta configuration is directly connected to a voltage source. As expected from having unbalanced impedances, we will see differences between the phases in current.

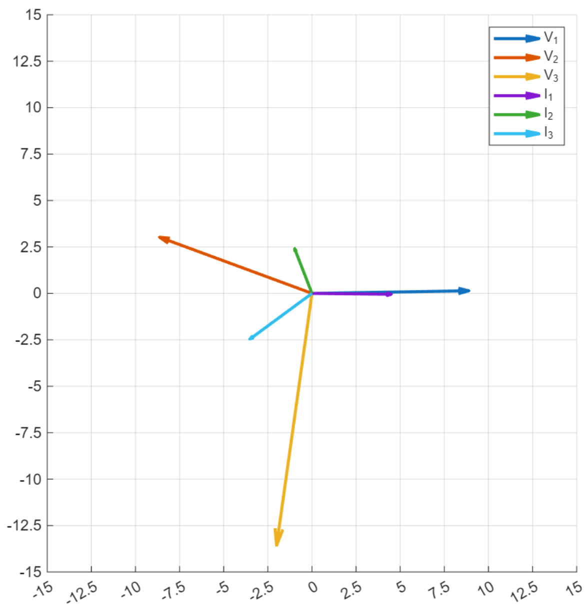


(a) Phasor diagram of circuit with RL balanced load, neutrals disconnected.

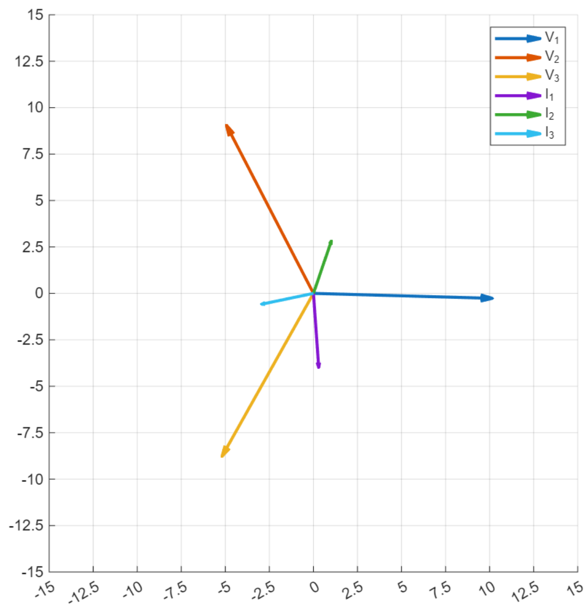


(b) Phasor diagram of circuit with RL unbalanced load, shorted resistor, neutrals disconnected.

Figure 5. Phasor plots of the applied phase voltages, V_1, V_2, V_3, and input currents, I_1, I_2, I_3, for the wye-connected RL load. Note that all current magnitudes have been scaled up by 10.

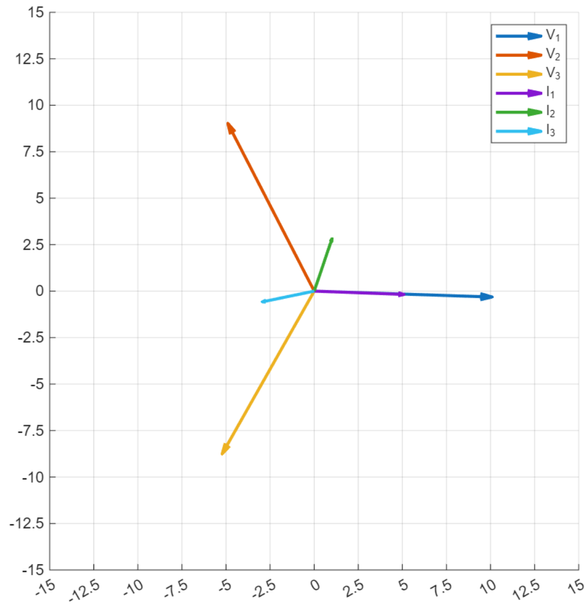


(a) Phasor diagram of circuit with RL unbalanced load, shorted inductor, neutrals disconnected.



(b) Phasor diagram of circuit with RL unbalanced load, shorted resistor, neutrals connected.

Figure 6. Phasor diagrams of the applied phase voltages, V_1, V_2, V_3, and input currents, I_1, I_2, I_3, for the wye-connected RL load. Note that all current magnitudes have been scaled up by 10.



(c) Phasor diagram of circuit with RL unbalanced load, shorted inductor, neutrals connected.

Figure 6. Phasor diagrams of the applied phase voltages, V_1, V_2, V_3, and input currents, I_1, I_2, I_3, for the wye-connected RL load. Note that all current magnitudes have been scaled up by 10.

Task 5D. Harmonics in AC Mains

For this section, we will use V_1 from Task 2B, with a balanced load.

Harmonics are distortions that occur at integer multiples of the fundamental frequency. For AC mains in North America, this is 60 Hz. These are caused by non-linear loads. In our lab, these harmonics may be caused by the power supply's internal components being non-linear. Since our load is purely ohmic, the load should not be contributing to the harmonic content.

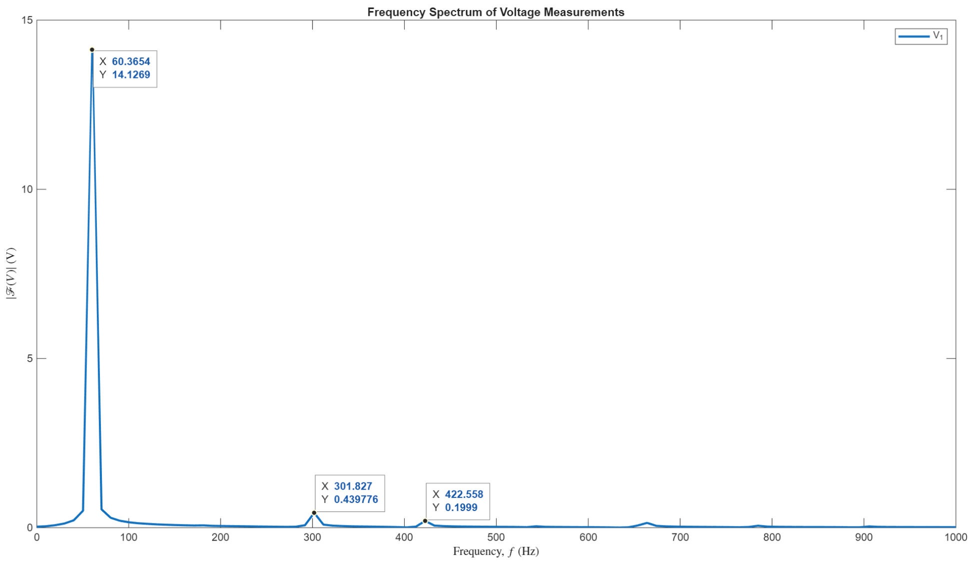


Figure 7. Frequency spectrum of V_1 from Task 2B using MATLAB's `fft()` function (see the appendix).

As expected, we see a large spike at 60 Hz as that is our power supply's frequency. We also notice distortions at multiples the fundamental frequency.

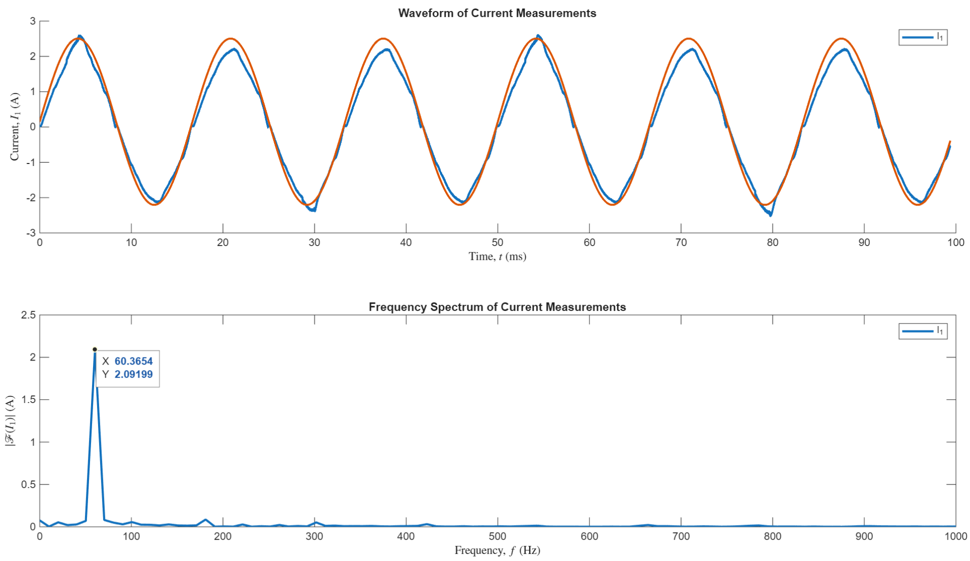
Task 5E. Single-Phase AC/DC

For this section, we will use I_1 from Task 3A, under a full load with and without a capacitor filter.

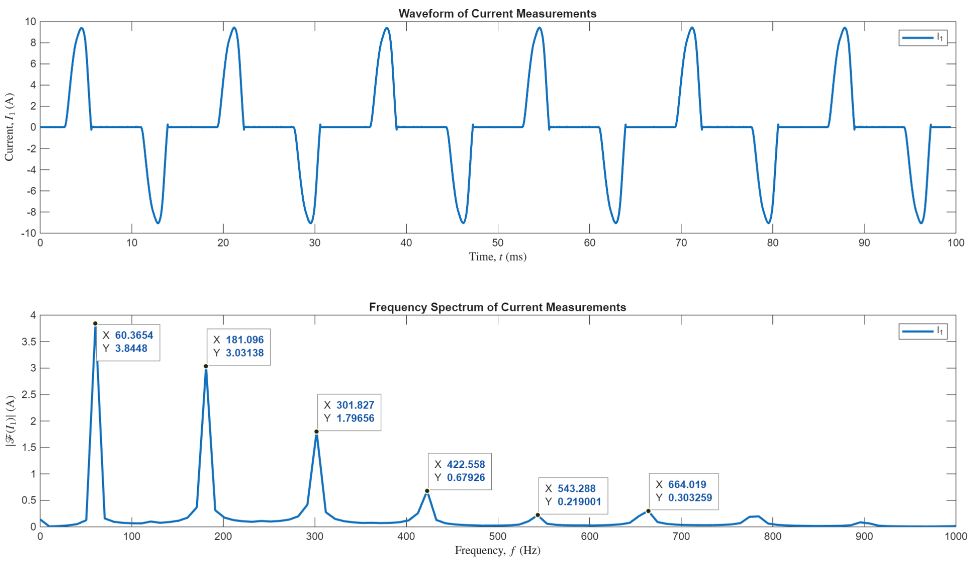
We see that the output current without a capacitor filter does look close to a sinusoidal wave. However, the peaks are sharper than a sine wave, resembling more of a sawtooth wave.

The output current with a capacitor filter, apart from the non-zero parts of the current, does not resemble a sinusoidal wave.

As expected from our initial observations on the waveforms, the frequency spectrum of the current without a capacitor filter has a single large spike at 60 Hz, with a small amount of distortion at higher frequencies. On the other hand, the current with a capacitor filter has strong harmonic content.



(a) Waveform and frequency spectrum under a full load without a capacitor filter.



(b) Waveform and frequency spectrum under a full load with a capacitor filter.

Figure 8. Waveform and frequency spectrum of I_1 from Task 3A using MATLAB's `fft()` function (see the appendix).

Task 5F. Three-Phase AC/DC Rectifiers

For this section, we will use I_1, I_2, and I_3 from Task 4A, under a full load with a capacitor filter.

The current I_2 is the output of the current from the full-bridge rectifier. The diodes ensure that all of the current flows in one single direction. For I_3, the capacitor helps maintain a steady voltage and current, making it comparable to an ideal DC current.

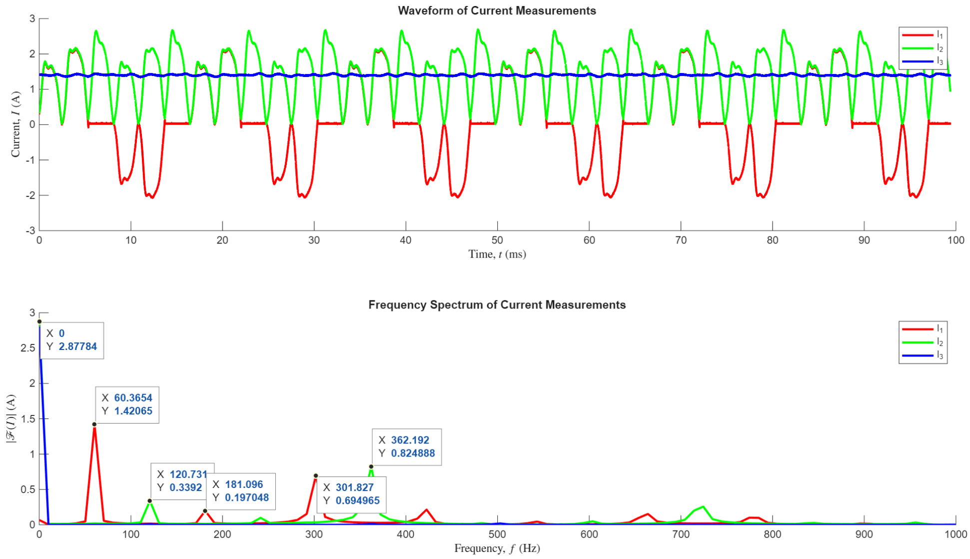


Figure 9. Waveform and frequency spectrum of I_1, I_2, and I_3 from Task 4A using MATLAB's `fft()` function (see the appendix).

The I_1 currents in Task 5E are much closer to an ideal sinusoidal wave, compared to here. However, we also observe that the three-phase input current has much less harmonic content compared to the single-phase currents.

Conclusion

In this laboratory experiment, we were able to measure and calculate different parameters of single-phase and three-phase circuits. We looked into real-world applications and ideal versus non-ideal components, such as inductors, capacitors, and rectifiers. We also saw the difference between balanced and unbalanced loads, and their different effects in the wye and delta configurations. In addition, this lab showed us the difference between single-phase and three-phase rectifiers. Finally, we looked into the harmonic content of AC voltages and currents.

Appendix A. Code Listings

```
function specan(filename)
	% Spectrum analyser
	%
	% Analyse and plot the frequency spectrum of a waveform
	% given by a tab-separated value (TSV) file.
	%
	% The following column headers for the TSV are assumed:
	% time, v1, v2, v3, i1, i2, i3
	%
	% Time measurements are assumed to be in milliseconds.
	% All voltages and currents are assumed to be given
	% in Volts and Amperes, respectively.

	data = readtable(filename, FileType="text", Delimiter="\t");

	L = length(data.time);
	% Assumes data is sampled at regular intervals.
	T = 1e-3*(data.time(end)-data.time(1))/(L-1);
	fs = 1/T;
	% Frequency axis for plotting.
	f = fs/L*(0:L-1);

	Y1 = abs(fft(data.v1)/L);
	Y2 = abs(fft(data.v2)/L);
	Y3 = abs(fft(data.v3)/L);
	Y4 = abs(fft(data.i1)/L);
	Y5 = abs(fft(data.i2)/L);
	Y6 = abs(fft(data.i3)/L);

	figure(1);

	subplot(2, 1, 1);
	hold on;
	plot(f, Y1, "r", LineWidth=2);
	plot(f, Y2, "g", LineWidth=2);
	plot(f, Y3, "b", LineWidth=2);
	title("Frequency Spectrum of Voltage Measurements");
	xlim([0 1e3]);
	xlabel("Frequency, $f$ (Hz)", Interpreter="latex");
	ylabel("$\left|\mathcal{F}(V)\right|$ (V)", Interpreter="latex");
	legend("V_1", "V_2", "V_3");

	subplot(2, 1, 2);
	hold on;
	plot(f, Y4, "r", LineWidth=2);
	plot(f, Y5, "g", LineWidth=2);
	plot(f, Y6, "b", LineWidth=2);
	title("Frequency Spectrum of Current Measurements");
	xlim([0 1e3]);
	xlabel("Frequency, $f$ (Hz)", Interpreter="latex");
	ylabel("$\left|\mathcal{F}(I)\right|$ (A)", Interpreter="latex");
	legend("I_1", "I_2", "I_3");
end
```

Listing 1. Frequency spectrum analysis code.

```
function poweran(filename, pf_angle)
        % Power analyser
        %
        % Analyse and plot the instantaneous, real, and reactive
        % power of a waveform given by a tab-separated value
        % (TSV) file.
        %
        % Required measurements to input:
        % power factor angle (degree)
        %
        % The following column headers for the TSV are assumed:
        % time, v1, v2, v3, i1, i2, i3
        %
        % Time measurements are assumed to be in milliseconds.
        % All voltages and currents are assumed to be given
        % in Volts and Amperes, respectively.

        data = readtable(filename, FileType="text", Delimiter="\t");

        L = length(data.time);

        t = linspace(0, data.time(end), L);
        p = data.v1(1:L) .* data.i1(1:L);

        % Sum over a full period to get the RMS values.
        n = length(data.time(data.time < 1e3/60));
        vrms = sqrt(sum(data.v1(data.time < 1e3/60).^2)/n);
        irms = sqrt(sum(data.i1(data.time < 1e3/60).^2)/n);

        % These are constant in time so we extrapolate with two points
        % to save compute time.
        real_power = ones(2) * vrms * irms * cos(deg2rad(pf_angle));
        reactive_power = ones(2) * vrms * irms * sin(deg2rad(pf_angle));

        display(vrms);
        display(irms);

        figure(1);

        subplot(3, 1, 1);
        plot(t, p, "r", LineWidth=2);
        title("Instantaneous Power versus Time");
        xlabel("Time, $t$ (ms)", Interpreter="latex");
        ylabel("Power, $p(t)$ (W)", Interpreter="latex");

        subplot(3, 1, 2);
        plot([0 data.time(end)], real_power, "g", LineWidth=2);
        title("Real Power versus Time");
        xlabel("Time, $t$ (ms)", Interpreter="latex");
        ylabel("Real Power, $P$ (W)", Interpreter="latex");

        subplot(3, 1, 3);
        plot([0 data.time(end)], reactive_power, "b", LineWidth=2);
        title("Reactive Power versus Time");
        xlabel("Time, $t$ (ms)", Interpreter="latex");
        ylabel("Reactive Power, $Q$ (VAR)", Interpreter="latex");
end
```

Listing 2. Power analysis code.

```matlab
function phasor(modulus, argument)
	% Wrapper for quiver() that plots phasor diagrams.
	%
	% Expects the modulus and argument (in degrees)
	% of a complex number.

	r = modulus;
	phi = argument * pi / 180;

	% Automatic scaling is disabled.
	quiver(0, 0, r*cos(phi), r*sin(phi), 0);

	xlim([-r r]);
	ylim([-r r]);
end
```

Listing 3. Phasor plot code.

```sh
#!/bin/sh

# Prepare all waveform data for MATLAB processing.
#
# This script will:
# 1.	sanitise the header before running MATLAB's readtable();
# 2.	convert line endings from DOS to UNIX; and
# 3.	rename files to have *.tsv file extensions.
#
# This script assumes all waveform data is tab-delimited and
# stored in *.txt files.
#
# Usage: ./prep.sh

files=$(ls -- *.txt)

for i in $files; do
	printf "Converting %s\n" "$i"

	# Replace the header and convert DOS line endings (CRLF)
	# to UNIX line endings (LF).
	sed -i -e "1c time\tv1\tv2\tv3\ti1\ti2\ti3" -e "s/\x1D$//" "$i"

	# Rename files to have TSV extensions.
	mv "$i" "${i%.*}.tsv"
done
```

Listing 4. Shell script for preparing data.

Appendix B. Images

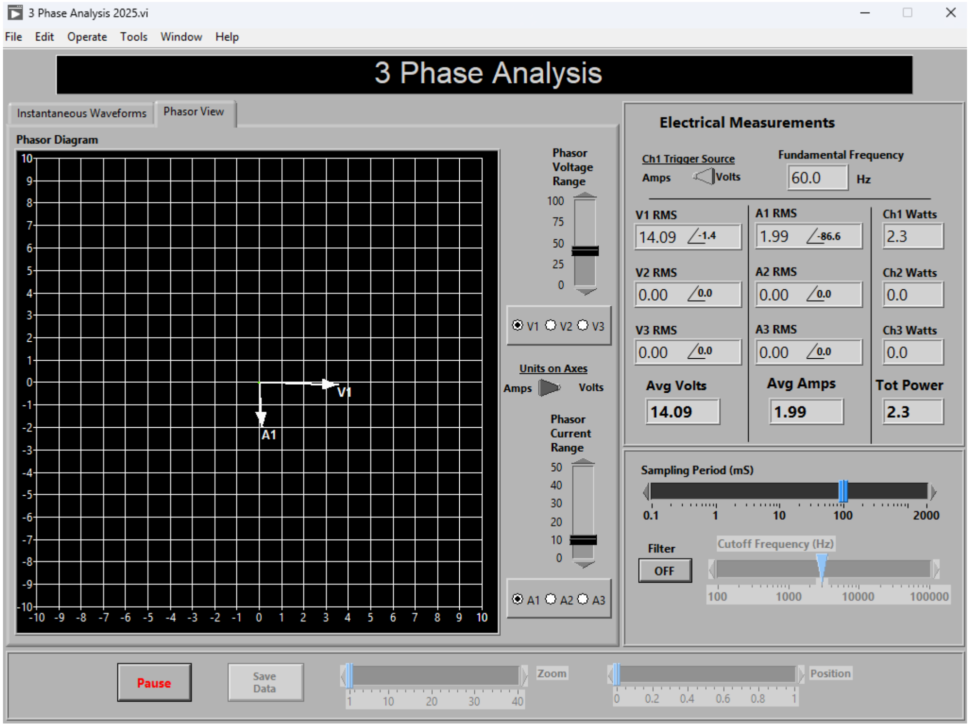


Figure 10. Phasor measurements for the RL load box with three inductors (Task 1A).

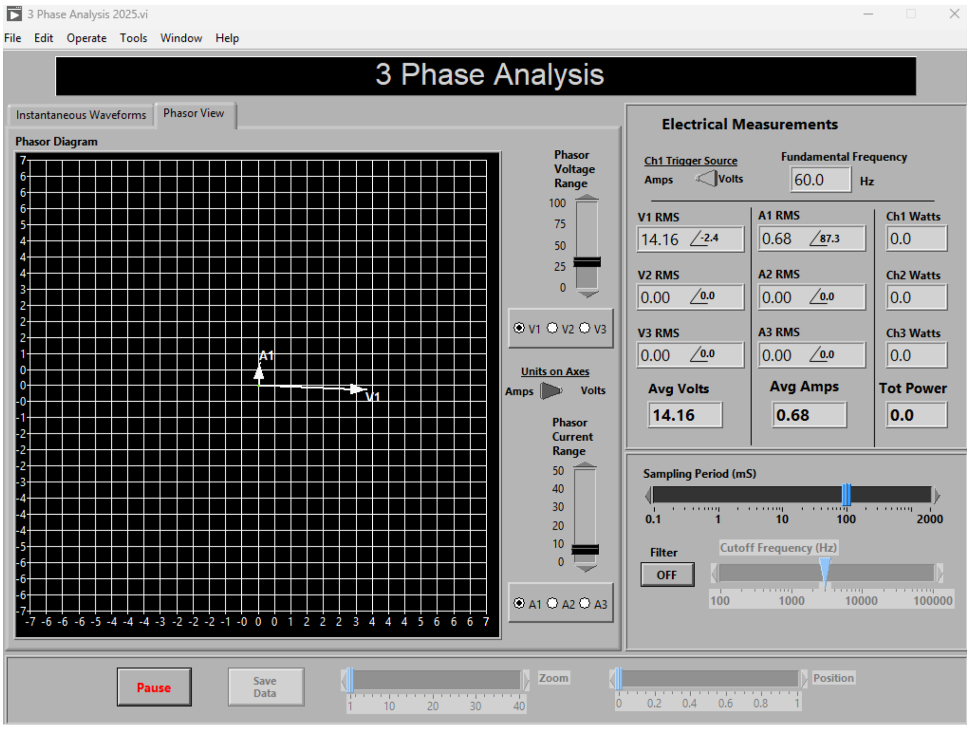
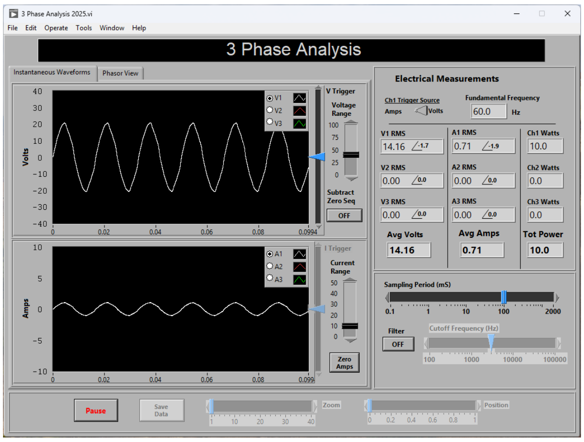
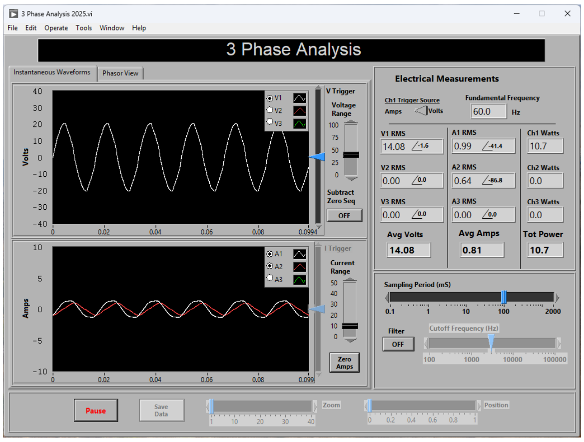


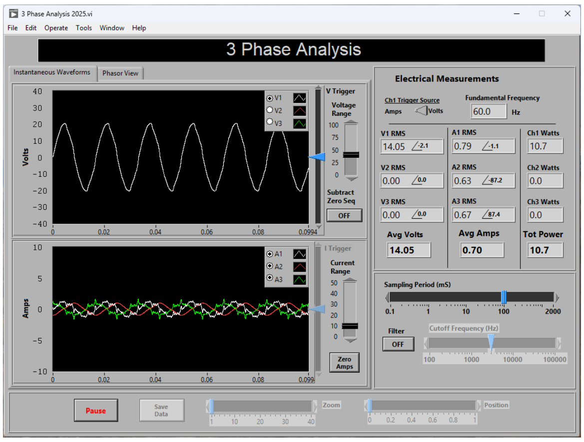
Figure 11. Phasor measurements for the RC load box with three capacitors (Task 1B).



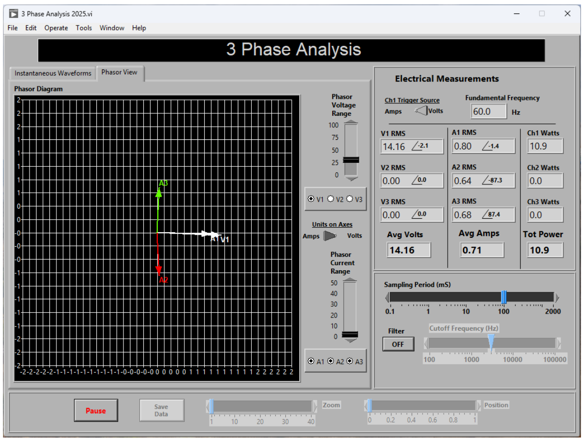
(a) Waveform of circuit with one resistor.



(b) Waveform of circuit with one resistor and one inductor.

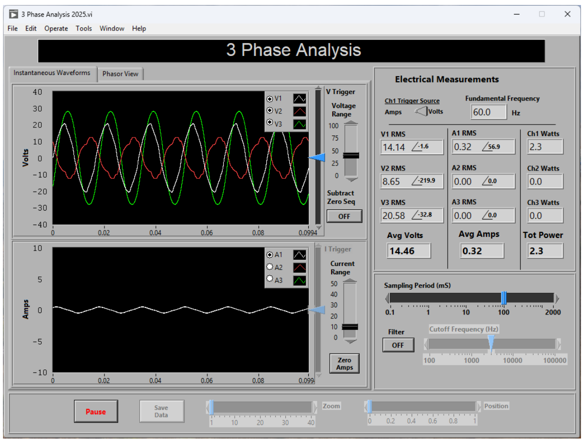


(c) Waveform of circuit with one resistor, one inductor, and three capacitors.

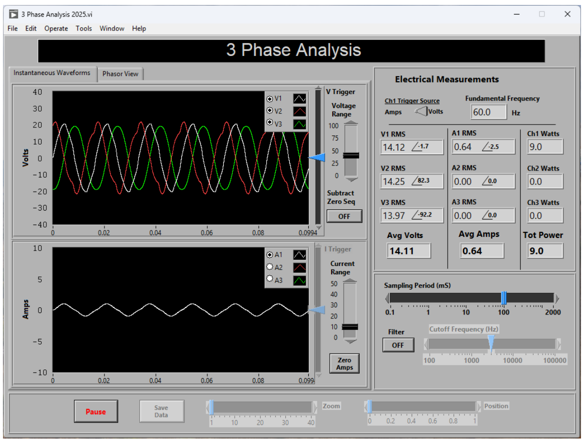


(d) Phasor view of circuit with one resistor, one inductor, and three capacitors.

Figure 12. Measurements of parallel connections of RLC components (Task 1C).

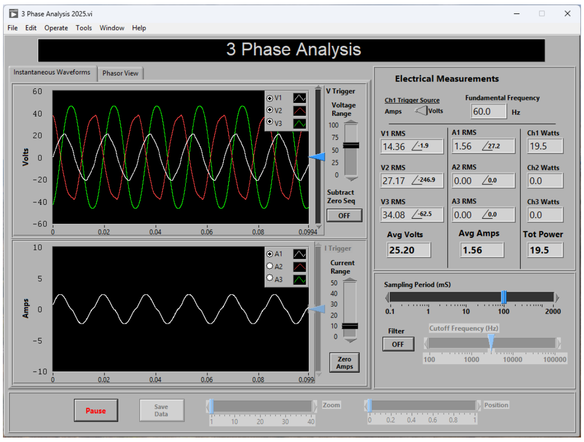


(a) Waveform of circuit with one resistor, one inductor, and one capacitor.

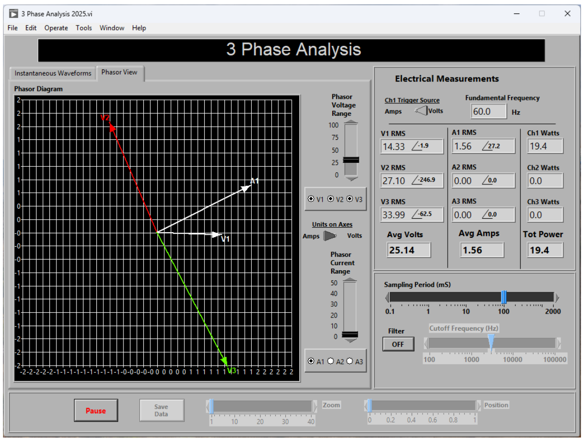


(b) Waveform of circuit with one resistor, one inductor, and three capacitors.

Figure 13. Measurements of series connections of RLC components (Task 1D).

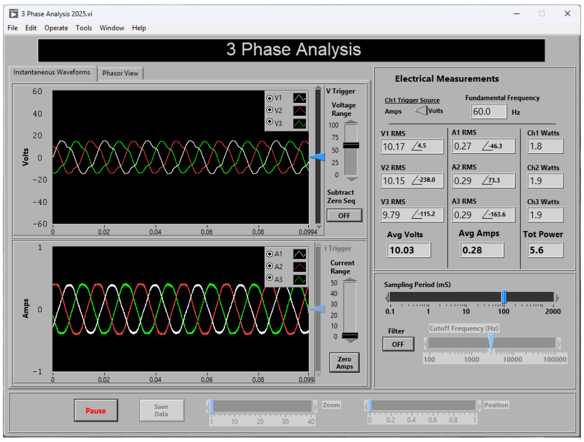


(c) Waveform of circuit with three resistors, one inductor, and three capacitors.

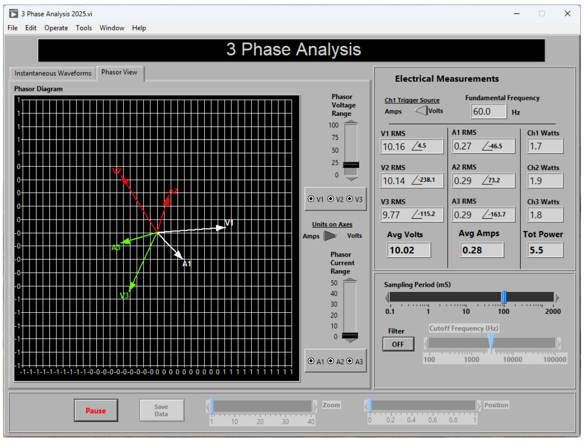


(d) Phasor view of circuit with three resistors, one inductor, and three capacitors.

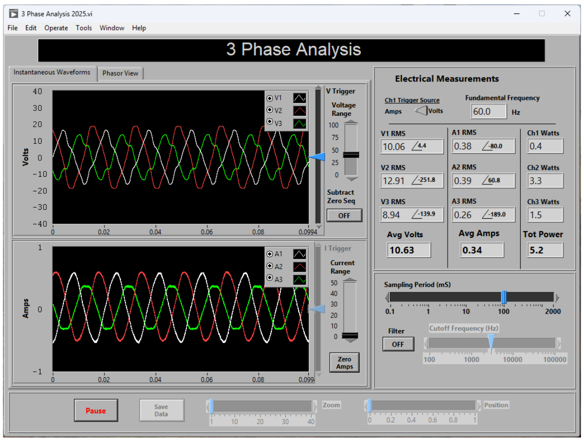
Figure 13. Measurements of series connections of RLC components (Task 1D).



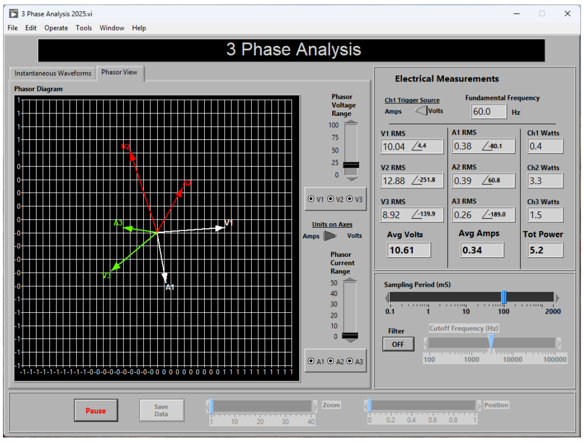
(a) Waveform of circuit with RL balanced load, neutrals disconnected.



(b) Phasor view of circuit with RL balanced load, neutrals disconnected.

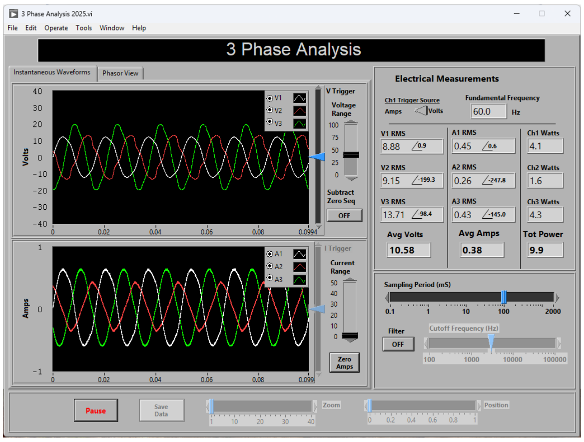


(c) Waveform of circuit with RL unbalanced load, shorted resistor, neutrals disconnected.

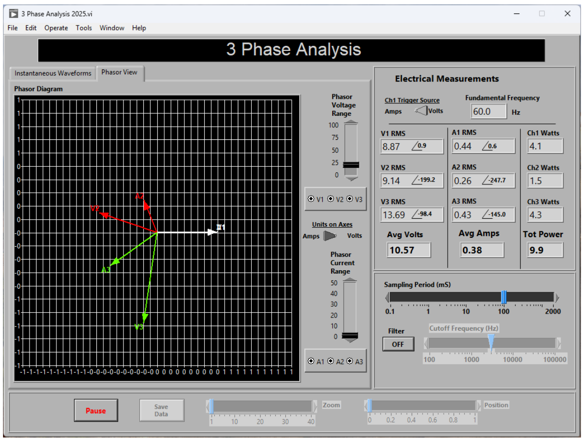


(d) Phasor view of circuit with RL unbalanced load, shorted resistor, neutrals disconnected.

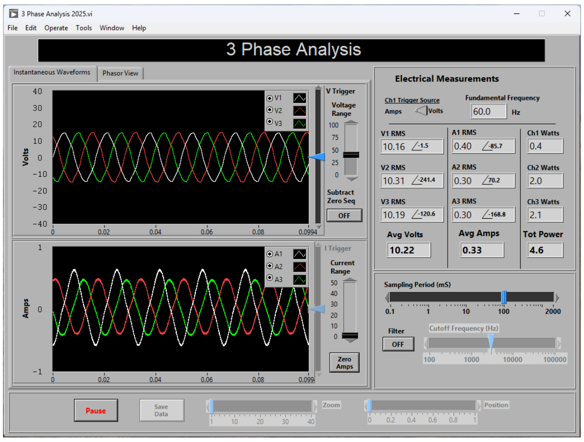
Figure 14. Three-phase measurements with wye-connected RL load box (Task 2A).



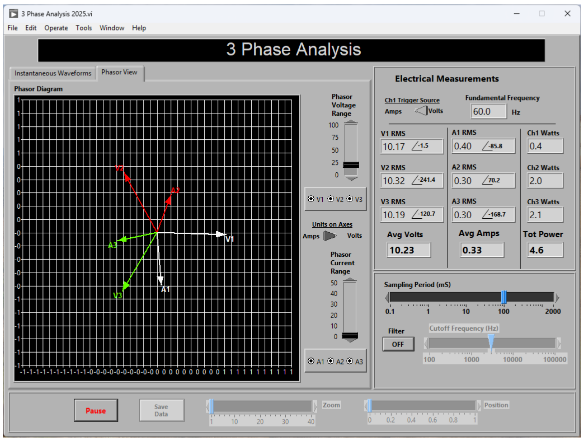
(e) Waveform of circuit with RL unbalanced load, shorted inductor, neutrals disconnected.



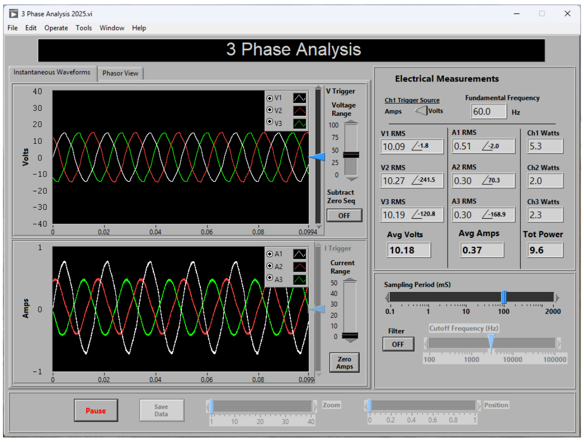
(f) Phasor view of circuit with RL unbalanced load, shorted inductor, neutrals disconnected.



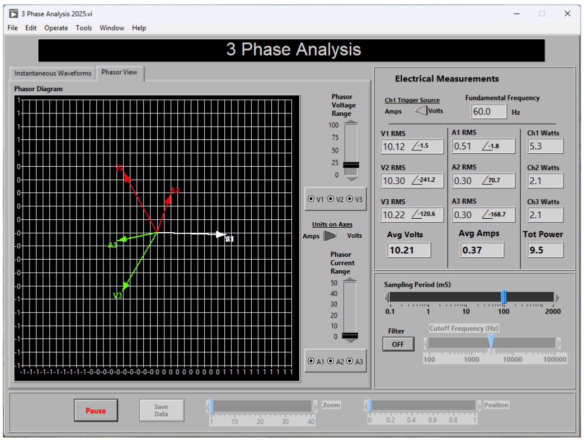
(g) Waveform of circuit with RL unbalanced load, shorted resistor, neutrals connected.



(h) Phasor view of circuit with RL unbalanced load, shorted resistor, neutrals connected.

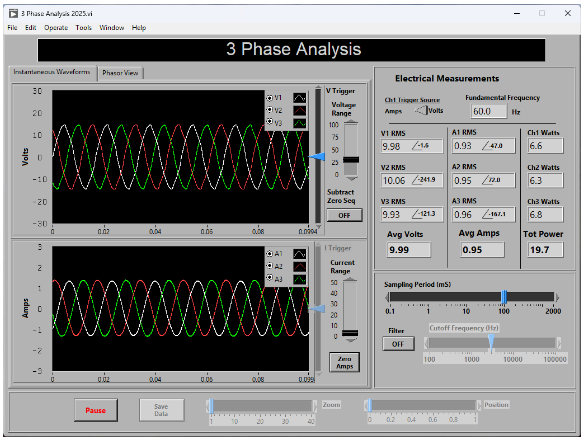


(i) Waveform of circuit with RL unbalanced load, shorted inductor, neutrals connected.

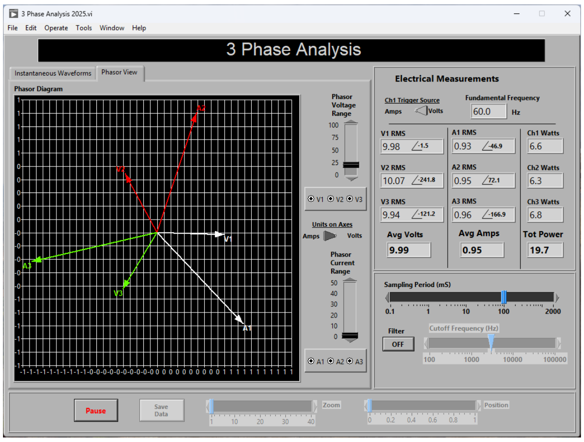


(j) Phasor view of circuit with RL unbalanced load, shorted inductor, neutrals connected.

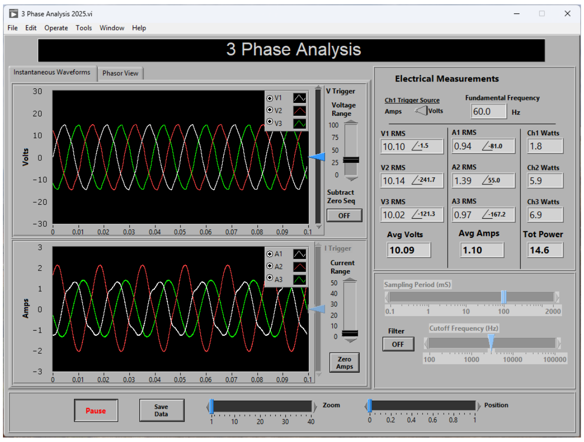
Figure 14. Three-phase measurements with wye-connected RL load box (Task 2A).



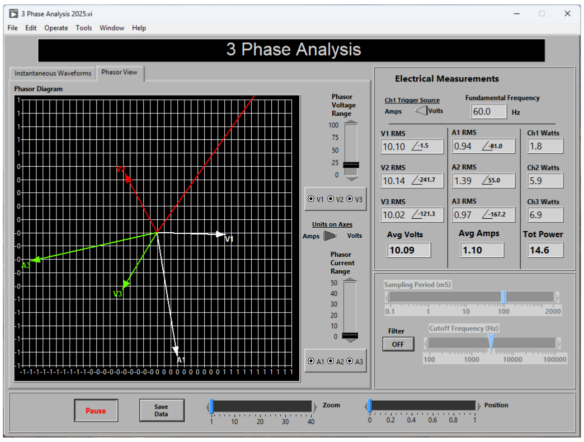
(a) Waveform of circuit with RL balanced load.



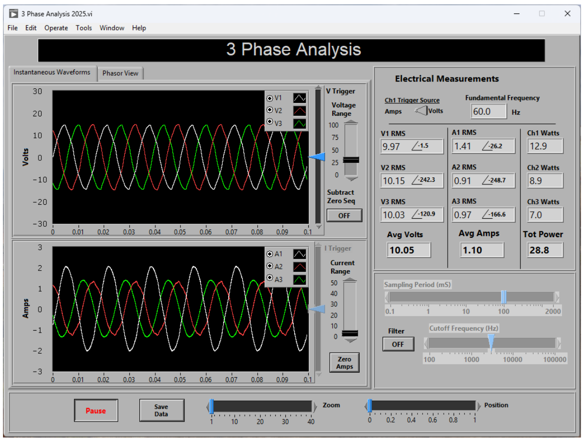
(b) Phasor view of circuit with RL balanced load.



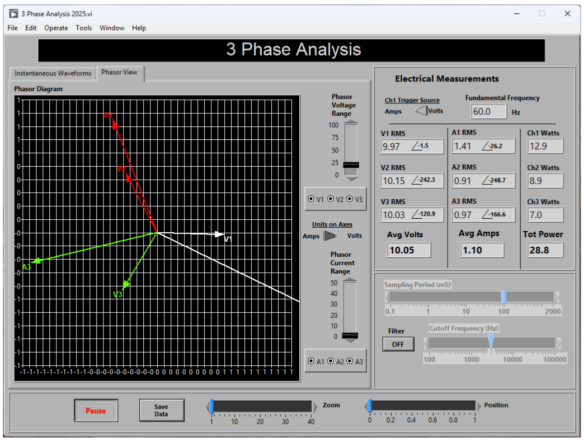
(c) Waveform of circuit with RL unbalanced load, shorted resistor.



(d) Phasor view of circuit with RL unbalanced load, shorted resistor.

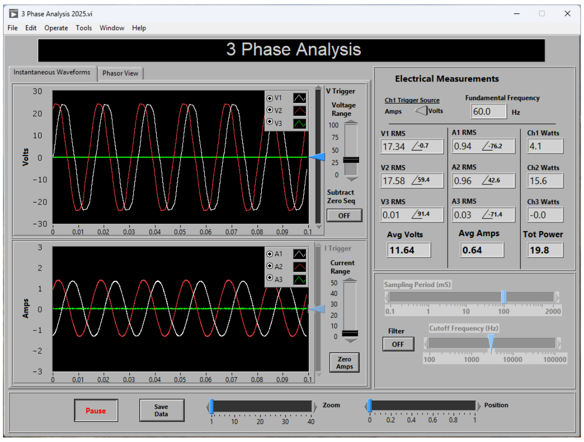


(e) Waveform of circuit with RL unbalanced load, shorted inductor.

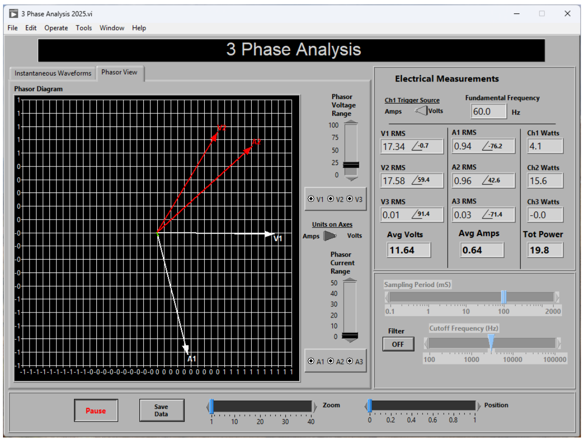


(f) Phasor view of circuit with RL unbalanced load, shorted inductor.

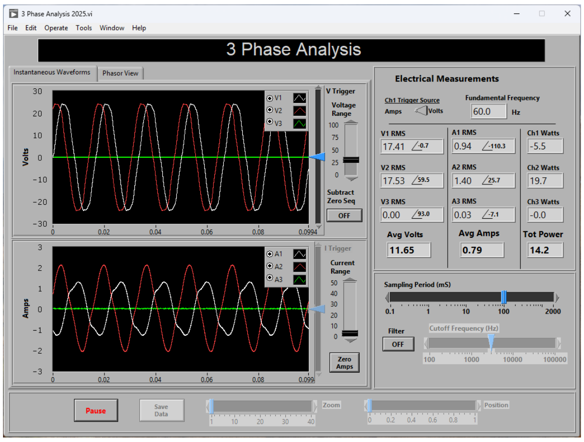
Figure 15. Three-phase measurements with delta-connected RL load box (Task 2B).



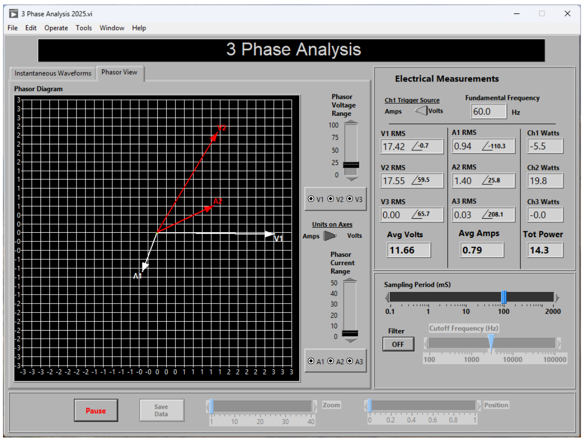
(a) Waveform of circuit with RL balanced load.



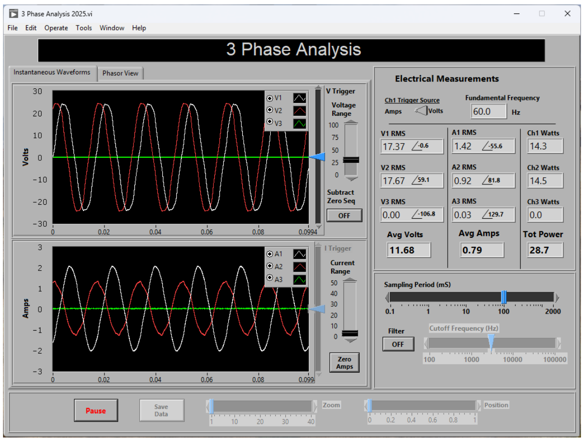
(b) Phasor view of circuit with RL balanced load.



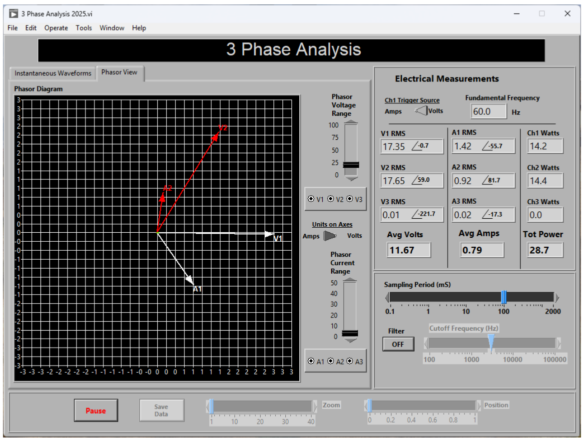
(c) Waveform of circuit with RL unbalanced load, shorted resistor.



(d) Phasor view of circuit with RL unbalanced load, shorted resistor.

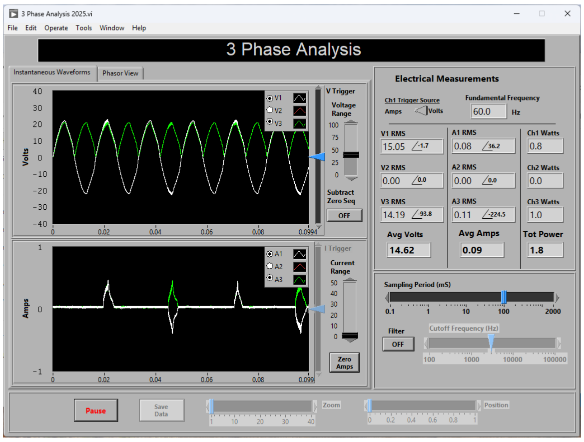


(e) Waveform of circuit with RL unbalanced load, shorted inductor.

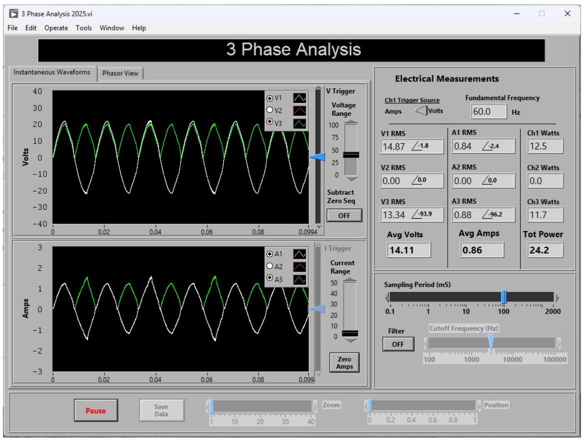


(f) Phasor view of circuit with RL unbalanced load, shorted inductor.

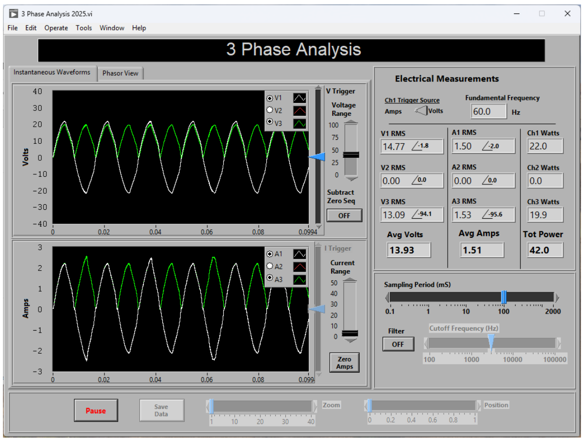
Figure 16. Two-wattmeter measurements with delta-connected RL load box (Task 2C).



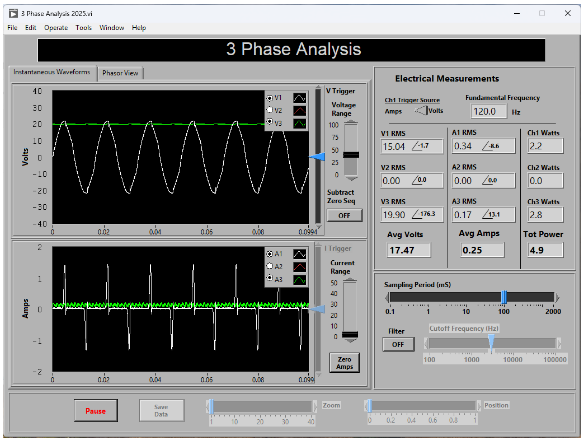
(a) Waveform of circuit without a capacitor filter under no load.



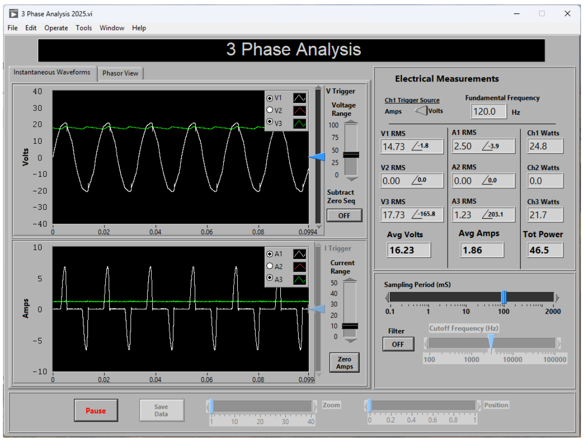
(b) Waveform of circuit without a capacitor filter under half load.



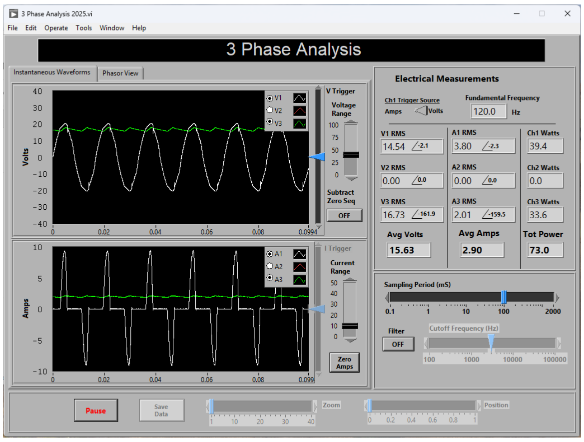
(c) Waveform of circuit without a capacitor filter under full load.



(d) Waveform of circuit with a capacitor filter under no load.

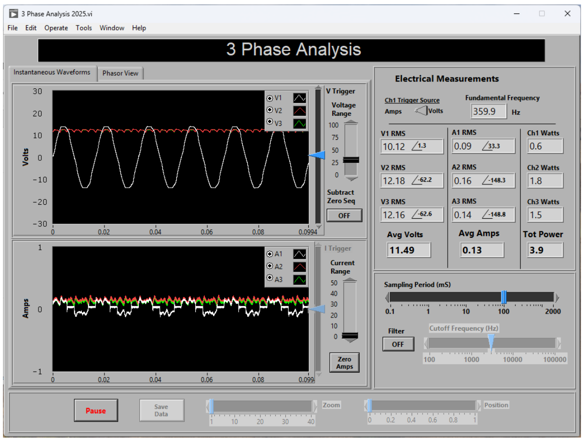


(e) Waveform of circuit with a capacitor filter under half load.

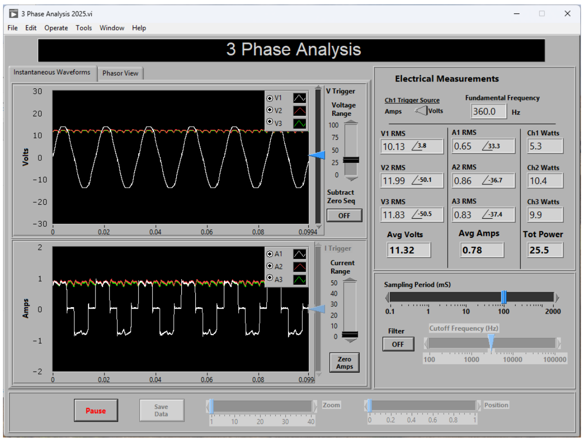


(f) Waveform of circuit with a capacitor filter under full load.

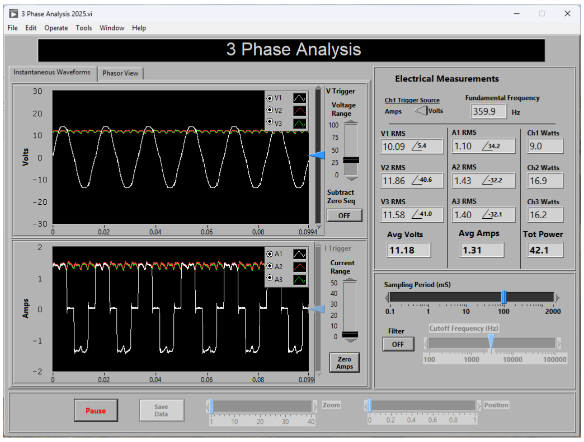
Figure 17. Single-phase full-wave rectifier, without and with capacitor filter (Task 3A).



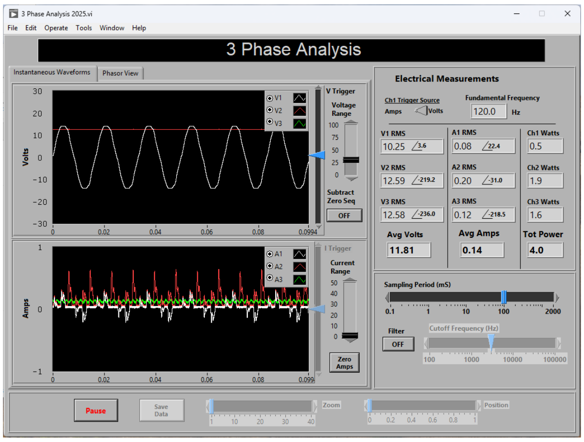
(a) Waveform of circuit without a capacitor filter under no load.



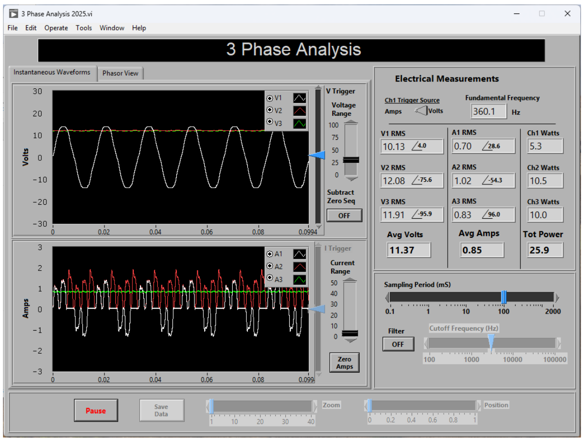
(b) Waveform of circuit without a capacitor filter under half load.



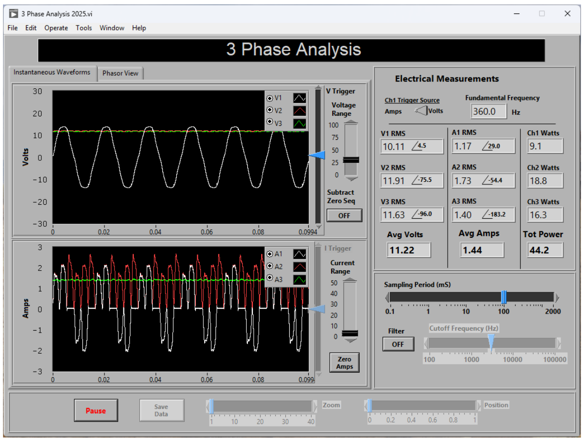
(c) Waveform of circuit without a capacitor filter under full load.



(d) Waveform of circuit with a capacitor filter under no load.



(e) Waveform of circuit with a capacitor filter under half load.



(f) Waveform of circuit with a capacitor filter under full load.

Figure 18. Three-phase full-wave rectifier, without and with capacitor filter (Task 4A).